ADDITIONAL MATHEMATICS
FORM 5
MODULE 4

INTEGRATION
CHAPTER 3 : INTEGRATION

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Indefinite Integral

\[ a \int a \, dx = ax + c. \]

\[ x^n \int dx = \frac{x^{n+1}}{n+1} + c. \]

The Equation of a Curve from Functions of Gradients

\[ y = \int f'(x) \, dx \]

\[ y = f(x) + c, \]
INTEGRATION

1. Integration is the **reverse process** of **differentiation**.

2. If \( y \) is a function of \( x \) and \( \frac{dy}{dx} = f'(x) \) then \( \int f'(x) \, dx = y + c, \quad c = \text{constant} \).

\[
\frac{dy}{dx} = f(x), \quad \int f(x) \, dx = y
\]

4.1. **Integration of Algebraic Functions**

**Indefinite Integral**

\( a \)  
\[
\int a \, dx = ax + c. \quad a \text{ and } c \text{ are constants}
\]

\( b \)  
\[
\int x^n \, dx = \frac{x^{n+1}}{n+1} + c. \quad c \text{ is constant, } n \text{ is an integer and } n \neq -1
\]

\( c \)  
\[
\int ax^n \, dx = a \int x^n \, dx = \frac{ax^{n+1}}{n+1} + c. \quad a \text{ and } c \text{ are constants } n \text{ is an integer and } n \neq -1
\]

\( d \)  
\[
\int [f(x) \pm g(x)] \, dx = \int f(x) \, dx \pm \int g(x) \, dx
\]
WORKED Example 1

Find the indefinite integral for each of the following.

\( a) \int 5dx \quad b) \int x^3dx \quad c) \int 2x^5dx \quad d) \int (x + 3x^2)dx \)

**Solution:**

\( a) \int 5dx = 5x + c \)
\( b) \int x^3dx = \frac{x^{3+1}}{3+1} + c = \frac{x^4}{4} + c \)
\( c) \int 2x^5dx = \frac{2x^{5+1}}{5+1} + c = \frac{2x^6}{6} + c = \frac{1}{3}x^6 + c \)
\( d) \int (x + 3x^2)dx = \int xdx + \int 3x^2dx = \frac{x^2}{2} + \frac{3x^3}{3} + c = \frac{x^2}{2} + x^3 + c \)

WORKED Example 2

Find the indefinite integral for each of the following.

\( a) \int \frac{x + 3x^2}{x^4}dx \quad b) \int x^2\left(3 - \frac{4}{x^2}\right)dx \)

**Solution:**

\( a) \int \frac{x + 3x^2}{x^4}dx = \int \left(\frac{x}{x^4} + \frac{3x^2}{x^4}\right)dx = \int \left(x^{-3} + 3x^{-2}\right)dx = \frac{x^{-2}}{-2} + 3\left(x^{-1}\right) + c = \frac{1}{2x^2} - \frac{3}{x} + c \)
\( b) \int x^2\left(3 - \frac{4}{x^2}\right)dx = \int \left(3x^2 - \frac{4}{x^2}\right)dx = \int \left(3x^2 - 4x^{-2}\right)dx = \frac{3x^3}{3} - 4\left(x^{-1}\right) + c = x^3 + \frac{4}{x} + c \)
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<td>3. Find $\int \left(2x + \frac{1}{x}\right)^2 , dx$.</td>
<td>4. Find $\int \left(2x^3 + x - \frac{3}{x^4} - 2\right) , dx$.</td>
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<td>7. Find $\int x^2 \left(6 + \frac{3}{x^8}\right) , dx$.</td>
<td>8. Integrate $\frac{x^2 + 3x + 2}{x + 1}$ with respect to $x$.</td>
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http://mathsmozac.blogspot.com
The Equation of a Curve from Functions of Gradients

If the gradient function of the curve is \( \frac{dy}{dx} = f'(x) \), then the equation of the curve is

\[
y = \int f'(x)\,dx \\
y = f(x) + c, \quad c \text{ is constant.}
\]

**WORKED Example 3**

Find the equation of the curve that has the gradient function \(3x - 2\) and passes through the point \((2, -3)\).

**Solution**

The gradient function is \(3x - 2\).

\[
\frac{dy}{dx} = 3x - 2 \\
y = \int (3x - 2)\,dx \\
y = \frac{3x^2}{2} - 2x + c
\]

The curve passes through the point \((2, -3)\).
Thus, \(x = 2, y = -3\).

\[-3 = \frac{3(2)^2}{2} - 2x + c\]
\[-3 = 6 - 4 + c\]
\[c = -5\]

Hence, the equation of curve is

\[
y = \frac{3x^2}{2} - 2x - 5
\]
EXERCISE B

1. Given that \( \frac{dy}{dx} = 6x - 2 \), express \( y \) in terms of \( x \) if \( y = 9 \) when \( x = 2 \).

2. Given the gradient function of a curve is \( 4x - 1 \). Find the equation of the curve if it passes through the point \((-1, 6)\).

3. The gradient function of a curve is given by \( \frac{dy}{dx} = kx - \frac{48}{x^3} \), where \( k \) is a constant.

Given that the tangent to the curve at the point \((-2, 14)\) is parallel to the \( x \)-axis, find the equation of the curve.
SPM 2003- Paper 2 :Question 3 (a)

Given that \( \frac{dy}{dx} = 2x + 2 \) and \( y = 6 \) when \( x = -1 \), find \( y \) in terms of \( x \). 

[3 marks]

SPM 2004- Paper 2 :Question 5(a)

The gradient function of a curve which passes through \( A(1, -12) \) is \( 3x^2 - 6x \). Find the equation of the curve. 

[3 marks]
SPM 2005- Paper 2 : Question 2

A curve has a gradient function \( px^2 - 4x \), where \( p \) is a constant. The tangent to the curve at the point \((1, 3)\) is parallel to the straight line \( y + x - 5 = 0 \).

Find

(a) the value of \( p \), \[3 \text{ marks}\]
(b) the equation of the curve. \[3 \text{ marks}\]
1. Find the indefinite integral for each of the following.

(a) \( \int (4x^3 + 3x - 2) \, dx \)

(b) \( \int \left( 3 - \frac{2}{x^2} + \frac{6}{x^3} \right) \, dx \)

(c) \( \int \left( \frac{2}{3} x^5 + \frac{1}{6x^5} \right) \, dx \)

(d) \( \int \left( \frac{x^2 + 3}{x^2} \right) \, dx \)

2. If \( \frac{dy}{dx} = 4x^3 - 4x \), and \( y = 0 \) when \( x = 2 \), find \( y \) in terms of \( x \).
3. If \( \frac{dp}{dv} = 2v - \frac{v^3}{2} \), and \( p = 0 \) when \( v = 0 \), find the value of \( p \) when \( v = 1 \).

4. Find the equation of the curve with gradient \( 2x^2 + 3x - 1 \), which passes through the origin.

5. Given that \( \frac{d^2y}{dx^2} = 4x \), and that \( \frac{dy}{dx} = 0 \), \( y = 2 \) when \( x = 0 \). Find \( \frac{dy}{dx} \) and \( y \) in terms of \( x \).
**EXERCISE A**

1) \( x^3 + 2x^2 - 10x + c \)
2) \( \frac{x^4}{2} - x^3 + 3x + c \)
3) \( \frac{4}{3} x^3 + 4x - \frac{1}{x} + c \)
4) \( \frac{x^4}{2} + \frac{x^2}{2} + \frac{1}{x^3} - 2x + c \)
5) \( -\frac{6}{x} - \frac{5}{2x^2} \)
6) \( \frac{x^2}{4} + \frac{2}{x} + c \)
7) \( 2x^3 - \frac{1}{x^3} + c \)
8) \( \frac{x^2}{2} + 2x + c \)

**SPM QUESTIONS**

1) \( y = x^2 + 2x + 7 \)
2) \( y = x^3 - 3x^2 - 10 \)
3) \( p = 3, \quad y = x^3 - 2x^2 + 4 \)

**ASSESSMENT**

1) \( a) \quad \frac{x^4}{2} + \frac{3}{2}x^2 - 2x + c \)
   
   \( b) \quad 3x + \frac{2}{x} - \frac{3}{x^2} + c \)
   
   \( c) \quad \frac{x^6}{9} - \frac{1}{24x^4} + c \)
   
   \( d) \quad \frac{x^3}{3} + 6x - \frac{9}{x} + c \)

2) \( y = x^4 - 2x^2 - 8 \)

3) \( p = \frac{7}{8} \)

4) \( y = \frac{2}{3}x^3 + \frac{3}{2}x^2 - x \)

5) \( y = \frac{2}{3}x^3 + 2 \)
ADDITIONAL MATHEMATICS
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MODULE 5
INTEGRATION
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CONCEPT MAP

INTEGRATION BY SUBSTITUTION

\[ \int (ax+b)^n \, dx = \int \frac{u^n}{a} \, du \]
where \( u = ax + b \),
a and \( b \) are constants, \( n \) is an integer and \( n \neq -1 \)

OR

\[ \int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c, \]
where \( a \), \( b \), and \( c \) are constants, \( n \) is integer and \( n \neq -1 \)

DEFINITE INTEGRALS

If \( \frac{d}{dx} g(x) = f(x) \) then

(a) \( \int_a^b f(x) \, dx = [g(x)]_a^b = g(b) - g(a) \)

(b) \( \int_a^b f(x) \, dx = -\int_a^b f(x) \, dx \)

(c) \( \int_a^b f(x) \, dx + \int_b^c f(x) \, dx = \int_a^c f(x) \, dx \)

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INTEGRATION BY SUBSTITUTION

\[ \int (ax+b)^n \, dx = \int \frac{u^n}{a} \, du \]
where \( u = ax + b, \) a and b are constants, n is an integer and \( n \neq -1 \)

OR

\[ \int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c, \]
where a, b, and c are constants, n is integer and \( n \neq -1 \)

SHARPEN YOUR SKILL

Find the indefinite integral for each of the following.

(a) \( \int (2x+1)^3 \, dx \)
(b) \( \int 4(3x+5)^7 \, dx \)
(c) \( \int \frac{2}{(5x+3)^3} \, dx \)

SOLUTION

(a) \( \int (2x+1)^3 \, dx \)

Let \( u = 2x + 1 \)

\[ \frac{du}{dx} = 2 \Rightarrow dx = \frac{du}{2} \]

\[ \int (2x+1)^3 \, dx = \int u^3 \left( \frac{du}{2} \right) \]

\[ = \int \frac{u^3}{2} \, du \]

\[ = \frac{u^4}{4} + c \]

\[ = \frac{u^4}{8} + c \]

Substitute \( u = 2x + 1 \)

Substitute \( 2x+1 \) and substitute \( dx \) with \( dx = \frac{du}{2} \)

\[ \int (2x+1)^3 \, dx = \frac{(2x+1)^4}{2(4)} + c = \frac{(2x+1)^4}{8} + c \]
(b) \[ \int (3x + 5)^7 \, dx \]
Let \( u = 3x + 5 \)
\[ \frac{du}{dx} = 3 \Rightarrow dx = \frac{du}{3} \]
\[ \int (3x + 5)^7 \, dx = \int \frac{4u^7}{3} \, du \]
\[ = \frac{4u^8}{3(8)} + c \]
\[ = \frac{u^8}{6} + c \]
\[ = \frac{(3u + 5)^8}{6} + c \]

OR
\[ \int (3x + 5)^7 \, dx = \frac{4(3x + 5)^8}{3(8)} + c \]
\[ = \frac{(3x + 5)^8}{6} + c \]

(c) \[ \int \frac{2}{(5x + 3)^3} \, dx = \int (5x + 3)^{-3} \, dx \]
Let \( u = 5x + 3 \)
\[ \frac{du}{dx} = 5 \Rightarrow dx = \frac{du}{5} \]
\[ \int (5x + 3)^{-3} \, dx = \int \frac{2u^{-3}}{5} \, du \]
\[ = \frac{2u^{-3}}{5(-2)} + c \]
\[ = \frac{u^{-2}}{-5} + c \]
\[ = -\frac{1}{5u^2} + c \]
\[ = -\frac{1}{5(5x + 3)^2} + c \]

DEFINITE INTEGRALS

If \( \frac{d}{dx} g(x) = f(x) \) then

(a) \[ \int_a^b f(x) \, dx = \left[ g(x) \right]_a^b = g(b) - g(a) \]

(b) \[ \int_a^b f(x) \, dx = -\int_a^b f(x) \, dx \]

(c) \[ \int_a^b f(x) \, dx + \int_a^c f(x) \, dx = \int_a^c f(x) \, dx \]
### SHARPEN YOUR SKILL

Evaluate each of the following

(a) \( \int_{1}^{2} \frac{(x+3)(x-3)}{x^4} \, dx \)

(b) \( \int_{0}^{1} \frac{1}{(2x+1)^2} \, dx \)

### SOLUTION

#### (a)

\[
\int_{1}^{2} \frac{(x+3)(x-3)}{x^4} \, dx + c = \int_{1}^{2} x^2 - 9x^{-4} \, dx
\]

\[
= \left. \frac{x^3}{3} - 9x^{-3} \right|_{1}^{2}
\]

\[
= \left( \frac{2^3}{3} - 9 \cdot 2^{-3} \right) - \left( \frac{1^3}{3} - 9 \cdot 1^{-3} \right)
\]

\[
= \frac{8}{3} - \frac{1}{2} - \left( \frac{1}{3} - 9 \right)
\]

\[
= \frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 9
\]

\[
= \frac{8}{3} - \frac{1}{2} - \frac{1}{3} + \frac{27}{3}
\]

\[
= \frac{1}{3}
\]

#### (b)

\[
\int_{0}^{1} \frac{1}{(2x+1)^2} \, dx = \int_{0}^{1} (2x+1)^{-2} \, dx
\]

\[
= \left. \frac{-1}{2(2x+1)} \right|_{0}^{1}
\]

\[
= \frac{-1}{2(2+1)} - \frac{-1}{2(0+1)}
\]

\[
= \frac{-1}{6} - \frac{-1}{2}
\]

\[
= \frac{1}{3}
\]
**INTEGRATE THE FOLLOWING USING SUBSTITUTION METHOD.**

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<td>(1)</td>
<td>$\int (x+1)^3 dx$</td>
</tr>
<tr>
<td>(2)</td>
<td>$\int -4(3x+5)^5 dx$</td>
</tr>
<tr>
<td>(3)</td>
<td>$\int \frac{1}{(5x+3)^3} dx$</td>
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<td>(4)</td>
<td>$\int \left( 5 - \frac{1}{2} x \right)^{-3} dx$</td>
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<tr>
<td>(5)</td>
<td>$\int 5 \left( 4 - \frac{1}{2} y \right)^4 dy$</td>
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<tr>
<td>(6)</td>
<td>$\int \frac{3}{2} \left( 5 - \frac{2}{3} u \right)^5 du$</td>
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### EXERCISE B

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<tr>
<td><strong>1. Evaluate</strong> ( \int_1^8 (x^3 + 4) , dx )</td>
<td><strong>2. Evaluate</strong> ( \int_{-3}^4 \frac{1}{8}x(x^2 - x + 5) , dx )</td>
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<td><strong>Answer</strong>: 1023.75</td>
<td><strong>Answer</strong>: ( \frac{83}{96} )</td>
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| **3. Integrate** \( \left( \frac{2}{3}x - 5 \right)^4 \) with respect to \( x \) | **4. Evaluate** \( \int_{-1}^1 \left( 2 - 3x + \frac{1}{x^2} \right) \, dx \) |
| **Answer**: \( \frac{3}{10} \left( \frac{2}{3}x - 5 \right)^5 + c \) | **Answer**: \( 3 \frac{1}{3} \) |

| **5. Evaluate** \( \int_1^3 \frac{(2x-1)(2x+1)}{4x^2} \, dx \) | **6. Given that** \( \int_2^5 f(x) \, dx = 10 \), find the value of \( \int_2^5 [1 - 2f(x)] \, dx \) |
| **Answer**: \( 1 \frac{5}{6} \) | **Answer**: 17 |
1. Given that \( \int_{1}^{2} f(x)dx = 3 \) and \( \int_{3}^{5} f(x)dx = -7 \). Find
   (a) the value of \( k \) if \( \int_{1}^{2} [kx - f(x)]dx = 8 \)
   (b) \( \int_{3}^{5} [5f(x) - 1]dx \)

   Answer: (a) \( k = \frac{22}{3} \)
   (b) 48

2. (a) \( \int \frac{4}{(2-3v)^6}dv \)
   (b) \( \int \frac{4}{3(1-5x)^3}dx \)

3. Show that \( \frac{d}{dx} \left( \frac{x^2}{3+2x} \right) = \frac{2x^2 + 6x}{(3+2x)^2} \).
   Hence, find the value of \( \int_{0}^{1} \frac{x(x+3)}{(3+2x)^2}dx \).

   Answer: \( \frac{1}{10} \)

4. Given that \( \int_{0}^{4} f(x)dx = 3 \) and \( \int_{0}^{4} g(x)dx = 5 \). Find
   (a) \( \int_{0}^{4} f(x)dx \cdot \int_{0}^{4} g(x)dx \)
   (b) \( \int_{0}^{4} [3f(x) - g(x)]dx \)

   Answer: (a) –15
   (b) 4
### SPM 2003 – PAPER 1, QUESTION 17

1. Given that \( \int 5 \frac{dx}{(1 + x)^3} = k (1 + x)^n + c \), find the value of \( k \) and \( n \) [3 marks]

Answer: \( k = -\frac{5}{3} \quad n = -3 \)

### SPM 2004 – PAPER 1, QUESTION 22

2. Given that \( \int_{-1}^{k} (2x - 3)dx = 6 \), where \( k > -1 \), find the value of \( k \). [4 marks]

Answer: \( k = 5 \)

### SPM 2005 – PAPER 1, QUESTION 21

3. Given that \( \int_{0}^{1} f(x)dx = 7 \) and \( \int_{0}^{1} (2f(x) - kx)dx = 10 \), find the value of \( k \).

Answer: \( k = \frac{1}{4} \)
### EXERCISE A

1. \(3 (x + 1)^4 + c\)

2. \(60 (3x + 5)^{-4} + c\)

3. \(-\frac{20}{(5x + 3)^3} + c\)

4. \(\frac{3}{2}(5 - \frac{1}{2}x)^{-2} + c\)

5. \(-10(4 - \frac{1}{2}y)^4 + c\)

6. \(-5\left(5 - \frac{2}{3}u\right)^6 + c\)

### EXERCISE B

1. 1023.75

2. \(\frac{83}{96}\)

3. \(\frac{3}{10}\left(\frac{2}{3}x - 5\right)^5 + c\)

4. \(3\frac{1}{3}\)

5. \(1\frac{5}{6}\)

6. 17

### ASSESSMENT

1. (a) \(k = \frac{22}{3}\)
   
   (b) 48

2. (a) \(90(2 - 3v)^{-5} + c\)
   
   (b) \(-\frac{100}{3}(1 - 5x)^{-4} + c\)

3. \(\frac{1}{10}\)

4. (a) \(-15\)
   
   (b) 4

### SPM QUESTIONS

1. \(k = -\frac{5}{3}\) \quad n = -3

2. \(k = 5\)

3. \(k = \frac{1}{4}\)
ADDITIONAL MATHEMATICS

MODULE 6

INTEGRATION
## CHAPTER 3: INTEGRATION

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Integration as a Summation

**AREA**

- a) The area under a curve which enclosed by x-axis, \( x = a \) and \( x = b \) is
  \[
  \int_a^b y \, dx
  \]
- b) The area under a curve which enclosed by y-axis, \( y = a \) and \( y = b \) is
  \[
  \int_a^b x \, dy
  \]
- c) The area enclosed by a curve and a straight line
  \[
  \int_a^b [f(x) - g(x)] \, dx
  \]

**VOLUME**

- a) The volume generated when a curve is rotated through 360° about the x-axis is
  \[
  V_x = \pi \int_a^b y^2 \, dx
  \]
- b) The volume generated when a curve is rotated through 360° about the y-axis is
  \[
  V_y = \pi \int_a^b x^2 \, dy
  \]
3. INTEGRATION

3.1 Integration as Summation of Area

The area under a curve which is enclosed by \( y = a \) and \( y = b \) is \( \int_{a}^{b} x dy \)

*Note:*
The area is preceded by a negative sign if the region is to the left of the \( y \)-axis.

The area enclosed by a curve and a straight line

The area of the shaded region = \( \int_{a}^{b} f(x)dx - \int_{a}^{b} g(x) \)

= \( \int_{a}^{b} [f(x) - g(x)]dx \)
1. Find the area of the shaded region in the diagram.

\[ y = x^2 - 2x \]

2. Find the area of the shaded region in the diagram.

\[ y = -x^2 + 3x + 4 \]
3. Find the area of the shaded region

4. Find the area of the shaded region in the diagram.
5. Find the area of the shaded region in the diagram.

6. Given that the area of the shaded region in the diagram above is \( \frac{28}{3} \) units\(^2\). Find the value of \( k \).
3.2 Integration as Summation of Volumes

The volume generated when a curve is rotated through 360° about the x-axis is

\[ V_x = \pi \int_a^b y^2 \, dx \]

The volume generated when a curve is rotated through 360° about the y-axis is

\[ V_y = \pi \int_a^b x^2 \, dy \]
Find the volume generated when the shaded region is rotated through 360° about the x-axis.

**Answer:**

Volume generated
\[ = \pi \int_{0}^{2} y^2 \, dx \]
\[ = \pi \int_{0}^{2} x^2 (x+1)^2 \, dx \]
\[ = \pi \int_{0}^{2} (x^4 + 2x^3 + x^2) \, dx \]
\[ = \pi \left[ \frac{x^5}{5} + \frac{2x^4}{4} + \frac{x^3}{3} \right]_{0}^{2} \]
\[ = \pi \left[ \left( \frac{2^5}{5} + \frac{2(2)^4}{4} + \frac{2^3}{3} \right) - 0 \right] \]
\[ = \frac{256}{15} \pi \]
\[ @ \quad 17 \frac{1}{15} \pi \ units^3 \]

The figure shows the shaded region that is enclosed by the curve \( y = 6 - x^2 \), the x-axis and the y-axis.

Calculate the volume generated when the shaded region is revolved through 360° about the y-axis.

**Answer:**

Given \( y = 6 - x^2 \)

Substitute \( x = 0 \) into \( y = 6 - x^2 \)

Then, \( y = 6 \)

Volume generated
\[ = \pi \int_{0}^{6} x^2 \, dy \]
\[ = \pi \int_{0}^{6} (6 - y) \, dx \]
\[ = \pi \left[ 6y - \frac{y^2}{2} \right]_{0}^{6} \]
\[ = \pi \left[ (6(6) - \frac{6^2}{2}) - 0 \right] \]
\[ = 18\pi \ units^3. \]
The above figure shows the shaded region that is enclosed by the curve $y = x (2 - x)$ and $x$-axis. Calculate the volume generated when the shaded region is revolved through $360^\circ$ about the $y$-axis. [4 marks]
2. The figure shows the curve $y = (x-2)^2$. Calculate the volume generated when the shaded region is revolved through $360^\circ$ about the $x$-axis.
The above figure shows part of the curve \( y = -\sqrt{3 - x} \) and the straight line \( x = k \). If the volume generated when the shaded region is revolved through 360° about the \( x \)-axis is \( \frac{11}{2} \pi \) units\(^3 \), find the value of \( k \).
Diagram 3 shows a curve \( x = y^2 - 1 \) which intersects the straight line \( 3y = 2x \) at point A.

Calculate the volume generated when the shaded region is involved \( 360^\circ \) about the y-axis.

[6 marks]
SPM 2004- Paper 2 : Question 10

Diagram 5 shows part of the curve \( y = \frac{3}{(2x - 1)^2} \) which passes through A(1, 3).

a) Find the equation of the tangent to the curve at the point A. [4 marks]

b) A region is bounded by the curve, the \( x \)-axis and the straight lines \( x = 2 \) and \( x = 3 \).
   i) Find the area of the region.
   ii) The region is revolved through 360º about the \( x \)-axis.
       Find the volume generated, in terms of \( \pi \). [6 marks]
In Diagram 4, the straight line PQ is normal to the curve \( y = \frac{x^2}{2} + 1 \) at A(2, 3). The straight line AR is parallel to the y-axis.

Find
\( \text{(a)} \) the value of \( k \), \hspace{1cm} [3 \text{ marks}]

\( \text{(b)} \) the area of the shaded region, \hspace{1cm} [4 \text{ marks}]

\( \text{(c)} \) the volume generated, in terms of \( \pi \), when the region bounded by the curve, the y-axis and the straight line \( y = 3 \) is revolved through 360° about y-axis. \hspace{1cm} [3 \text{ marks}]
### EXERCISE A

1. \( \frac{1\frac{1}{3}}{} \) units\(^2\)

2. \( \frac{20\frac{5}{6}}{} \) units\(^2\)

3. \( \frac{2\frac{2}{3}}{} \) units\(^2\)

4. \( \frac{24\frac{2}{3}}{} \) units\(^2\)

5. \( \frac{1}{2} \) units\(^2\)

6. \( k = 4 \)

### EXERCISE B

1. \( \frac{1\frac{1}{15}}{} \) unit\(^2\)

2. \( \frac{6\frac{3}{5}}{} \) unit\(^3\)

3. \( k = -2 \)

### SPM QUESTIONS

**SPM 2003**

Volume Generated = \( \frac{52}{15} \pi \) units\(^3\)

**SPM 2004**

1) Area = \( \frac{1}{5} \) units\(^2\)

ii) Volume Generated = \( \frac{49}{1125} \pi \) units\(^3\)

**SPM 2005**

a) \( k = 8 \)

b) Area = \( 12\frac{1}{3} \) units\(^2\)

c) Volume Generated = \( 4\pi \) units\(^3\)